

GETTING STARTED

1

CHAPTER

*Make everything
as simple as possible,
but not simpler.*
– Albert Einstein

This chapter is designed to introduce you to some ideas about mathematical processing. It should help you become comfortable with the displays we will be using and with some straightforward calculations. Moreover, it will take you briefly to the outer limits of contemporary mathematical thinking.

1.1 Demonstrations

This text is almost completely self-contained. You will participate in activities that demonstrate mathematical ideas and you will have the opportunity to work many exercises often interacting directly with on-screen programmed graphics in what we call panels. In each instance you will receive explicit instructions how to proceed. Thus all you will need to bring to your study of this book are paper and pencil to complete written exercises.

The panel demonstrations will give you access to mathematical concepts and it is important to your own understanding that you use them effectively. Much effort has been devoted to make these graphics easily accessible to you. Be thankful that you do not have to pay attention to their design because many hours were devoted to their development. Does that mean that you can relax and nod off as the text does your work? No indeed. You'll still need to be alert. In no way will the demonstrations take over for your brain;⁷ rather, they should help you focus your attention on the mathematical concepts rather than the complex computations they avoid.

These demonstrations serve a role similar to that of a word processor. When you type a message using a word processor like Word, it takes care of tasks like moving to new lines, spacing and indenting paragraphs, and checking spelling. On

⁷In Jonathan Swift's *Gulliver's Travels*, Gulliver visits an Academy where it has been discovered that content can be conveyed to students via their digestive systems. Students are fed specially developed wafers to absorb the ideas written on them. Unfortunately, as Gulliver tells us, the perverse students spit out the wafers.

a typewriter you had to do all those things yourself. You don't have to see how those tasks are completed by a word processor unless you study them in computer science courses. Just so here, you need not worry how activities were programmed or how diagrams are drawn. Instead, you need only concentrate on working with the displays and interpreting the results of what they demonstrate.

What I am trying to accomplish through these easily accessible displays is allow you to circumvent any dislike or fear of mathematics that you harbor. Such emotions usually arise from your getting bogged down in details and missing the main ideas. Here I aim to carry you past those details.

So I invite you to explore two of these demonstrations in these first exercises.

Exercises 1.1

(1.1.1) To get you started and to provide an interesting introductory experience with figures produced through use of trigonometry, I invite you to do some experimenting. Click to call up Panel 1.1.1, enter 5 and hit GO. Your display should produce a "wheel" of five regular five-sided polygons called pentagons. The adjective "regular" means that all sides and angles are equal. Before you move on, I urge you simply to stare at this display for a few moments to admire its symmetries. It only involves two kinds of geometric elements: line segments and angles that are calculated by the program in the background.

Panel 1.1.1: A Wheel of Regular Polygons



(1.1.2) Next enter 6 and GO in Panel 1.1.1. Although this leads to the same kind of display for regular six-sided polygons or hexagons, you should notice more about it. There is an optical illusion here. Can you see it? What look like cubes in the figure overlap and you should try to find two different ways of looking at them. If you have trouble, focus on one of the cube corners and see if it doesn't change position. Not everyone can find these differences and you may wish to compare notes with another person viewing the same display.

(1.1.3) Now I invite you to do some experimenting. Run Panel 1.1.1 with different numbers and observe what results. Determine your personal favorite among the displays. There is, of course, no correct answer. Include among your trials some large numbers. (The panel will accept large numbers, but best stick to 25 or less.) Do the results for large numbers remind you of the toy called a slinky? There is no hole in the center of these figures, but a donut shape (with or without a hole) is called a torus in mathematical jargon. The next time you visit a bakery, you can upset the proprietor by asking for a dozen powdered toruses.

(1.1.4) Okay, those exercises were designed to give you (finally, after all these years) a pleasant experience with a math-related display. Now I ask you to do some thinking related to a similar display. Run Panel 1.1.2 for values from 2 through 8, in each case recording the number of bricks in the display. Try to come up with a relationship

between the number you entered, n , and the number of bricks, b , that result. You may do this with an equation like $b = 3n$ or you may do so in your own words, like “The number of bricks is three times the number I entered.” (That is not, of course, the right answer.) This isn’t an easy task; look for patterns if you don’t find an answer quickly.

Panel 1.1.2: Brick Walls



(1.1.5) Show these displays to a friend. What are their reactions?

1.2 Recent Computing History

There are still old fogies like me who were around before electronic calculators were available. The very first were introduced in the 1970s and those were what we now call four bangers, that is, calculators that process only the four elementary arithmetic operations: addition, subtraction, multiplication and division. Very quickly additional power was added. Scientific calculators were soon introduced that carry out math functions — sines, cosines, powers, roots, logs and exponents — followed shortly by programmable calculators, calculators that support graphing and other features, and now finally iPhones with apps that carry out all these activities.



Figure 1.2.1 A standard office adding machine until the 1970s

Before such calculators were available, we used desktop machines, many of which were as big as today’s toaster ovens. With most of them we could only add and subtract; the few that did allow multiplication were more complicated. The machines were not electronic and you had to hand-crank into them each entry whose digits you had typed on the big keyboard.⁸ Given our experience, we oldsters appreciate the power that the new computation gives us more than you who have had hand-held calculators and smart phone apps that duplicate that processing through your entire lives.

⁸If, for example, you wanted to multiply 634×3 you would crank in 634 three times, thus converting multiplication to multiple addition: $634 \times 3 = 634 + 634 + 634$.

Now those sophisticated calculators have become so widespread that today they are like coffee table books, something to have around to show that you are educated — but seldom, if ever, are their advanced functions used. And even those that do get used are applied to problems that hardly deserve using a calculator at all. For example, when I am doing a series of calculations I occasionally, when addressing more demanding problems, find myself using a calculator to find products like 7×8 .

You'll need to have your own scientific calculator or phone scientific calculator app as you progress through this text. You'll soon see that even inexpensive calculators are powerful tools and hopefully you'll soon have yours tamed and performing interesting tricks. As you now begin to use any calculator for serious tasks, you should also begin thinking about how it can (and occasionally cannot) work for you. The same is true of this book.

Order of Operations

Today almost all calculations follow what has come to be called algebraic order. The rules are:

1. Perform operations in parentheses, innermost parentheses first.⁹
2. Perform exponents and roots.
3. Perform multiplication and division working from left to right.
4. Perform addition and subtraction working from left to right.

Following these rules of order you have this kind of step by step evaluation:

$$\begin{aligned}8 + 5(60 - 7^2) &= \\8 + 5(60 - 49) &= \\8 + 5(11) &= \\8 + 55 &= \\63 &= \end{aligned}$$

Algebraic order was not always followed by early calculators and computers and even today there are a few exceptions. Some simply calculated left to right, a few from right to left, and still others by Reverse Polish or Lukasiewicz order, a mix of the two that saved computer memory allocation.

⁹Some symbols other than the usual (and) symbols also play the role of parentheses. These include the bar (vinculum) in a radical like $\sqrt{a + b}$, which is equivalent to the $\sqrt{(a + b)}$ you would key on a calculator to represent this value. That bar also appears in fractions. Both numerator and denominator of fractions are considered as though they are in parentheses. You see this when you write a fraction in horizontal form as, for example, $\frac{a+b}{c+d} = (a + b)/(c + d)$.

Exercises 1.2

(1.2.1) In the order of operations example cited in the text, indicate which of the four rules was applied at each equals sign.

(1.2.2) There are two ways to calculate $1/273$, the reciprocal of 273 with a calculator:

Method 1. Find $1 \div 273$

Method 2. Use the x^{-1} key. (Key 273, then the x^{-1} key.) Your answers by Method 1 and Method 2 should, of course, be the same. Note that this is another use of the minus sign. Here the exponent -1 represents the inverse.

Formally the reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$. Thus $(\frac{a}{b})^{-1} = \frac{b}{a}$. What then should be $((\frac{a}{b})^{-1})^{-1}$? Check this by calculation.

(1.2.3) Many of the keys on a calculator have their inverses as the 2nd key. An inverse operation “undoes” the action of the operator, just as division by 5 “undoes” multiplication by 5. The following exercises should clarify this idea. Be sure you use the 2nd key to get the inverse.

- Use the x^2 key to calculate 273^2 . (Key 273 then x^2 .)
- Find the square root of your result in (a), that is, $\sqrt{273^2}$. (Key 273 then x^2 , then 2nd x^2 .)
- Calculate $(\sqrt{273})^2$
- Calculate $(-273)^2$. To key negative numbers like -273 , use the negative key, $(-)$, not the subtraction key, $-$.
- Find the square root of your result in (d), that is, $\sqrt{(-273)^2}$.
- Try to calculate $\sqrt{-273}$. What went wrong?
- Calculate $\log(10^{2.76})$
- Calculate $10^{\log 7.5}$
- Calculate $\ln(e^{\ln 5})$
- Calculate $\ln(e^{12.27})$
- Calculate $\cos^{-1}(\cos 30^\circ)$ (Be sure your calculator is in DEGREE mode.) The \cos^{-1} function, is the function that undoes cosine.
- Calculate $\cos(\cos^{-1} .5)$
- Calculate $\cos^{-1}(\cos 225^\circ)$ Although you get what may be an unexpected answer, check to see that $\cos 225^\circ$ and the cosine of your answer are the same. This is another of those mathematical concepts that seems designed to confuse non-specialists. As in exercise (e), mathematicians apply special rules to achieve such results. One reason for this seeming madness: mathematicians wanted to provide tables for only trigonometric functions between 0° and 90° .

(1.2.4) Evaluate each of the following without using your calculator and then check your answers with your calculator:

(a) $2 + 3 \times 5^2$

(c) $[(2 + 3)5]^2$

(e) $3^2 - 2^3$

(b) $(2 + 3)5^2$

(d) $\frac{5}{(2+3)^2}$

(f) $\sqrt{2+7} - \sqrt{4+3 \times 7}$

(1.2.5) The evaluation order for so-called towers of powers are from the top down. For example, $5^{2^3} = 5^8 = 390625$. Because this notation is difficult to represent, it is occasionally abbreviated using the symbol, \uparrow , so $5^{2^3} = 5 \uparrow 2 \uparrow 3$. Use your calculator to evaluate the following (if possible):

(a) $2 \uparrow 2 \uparrow 2$

(b) $2 \uparrow 2 \uparrow 2 \uparrow 2$

(c) $2 \uparrow 2 \uparrow 2 \uparrow 2 \uparrow 2$

(d) The answer to (c) is a number with 19,729 digits. You cannot calculate and record this result easily, but you can find its units digit if you look for a pattern among the powers. What is it and why?

(1.2.6) In the past computation was a necessary skill. To run a grocery or manage a store you had to calculate total sales and figure taxes. Today those paper-and-pencil tasks are done by electronic cash registers. But there are still situations when you are left to perform those calculations. One of those is when you pay a restaurant waiter and wish to include an appropriate tip. Panel 1.2.1 does the calculation of tax and tip for any purchase.

Panel 1.2.1. Calculating a Restaurant Payment



- (a) Run the program for a few values using your local sales tax and whatever tip percent you wish to use.
- (b) Does the program figure the amount of the tip in the original cost or on the cost after the tax is added?
- (c) How did you determine your answer to (b)?

(1.2.7) You have \$20 to spend on a meal. By experimenting with the display determine the maximum menu price you can afford if you add a tax of 7% and a tip of 15%. Suggestion: you might start by guessing \$19.

(1.2.8) Now you have a program that will do these simple calculations for you. (You may have had one already in your phone.) But, although you may have seen people using such programs for this purpose in restaurants, most people would be embarrassed to do so. What are some ways that you might estimate:

(a) A 20% tip?

(b) A 15% tip?

1.3 Order and disorder

*Clouds are not spheres, mountains are not cones,
coastlines are not circles, and bark is not smooth,
nor does lightning travel in a straight line.*
– Benoit Mandelbrot, fractal pioneer

In this section I introduce two seemingly contradictory aspects of mathematical calculations: order and disorder. Remarkably, they seem almost like the opposite faces of the same coin. First, consider chaotic events.

Disorder

I return to the question about that seemingly unnecessary accuracy of 10 or more digits. There is another reason for such accuracy that is suggested by an experiment designed by Edward Burger and Michael Starbird.¹⁰ They explore what happens when you carry out the following simple process:

1. Begin with .5.
2. Square the result and subtract 2.
3. Repeat step 2 again and again.

Beginning with that .5, the first time through this algorithm gives you $.5^2 - 2 = .25 - 2 = -1.75$. The second time (using that -1.75 gives: $(-1.75)^2 - 2 = 3.0625 - 2 = 1.0625$. Here is a listing of the first ten values you would obtain:

Iteration	Algorithm results
1	-1.75
2	1.0625
3	-.87109375
4	-1.241195679
5	-.4594332871
6	-.1788921055
7	1.200238540
8	-.5594274476
9	-1.687040931
10	.8461071025

Carrying out a repeated process like that is called iteration — that is mathematical jargon for repeating over and over. In this case the iteration replaces x with $x^2 - 2$. This is designated $x^2 - 2 \rightarrow x$ and read “ x squared minus 2 store x ,” or “replace x with x squared minus 2.”

Most scientific calculators carry out calculations to about 12 digit accuracy and report them on the calculator screen to 10 digits. For that reason the values in that

¹⁰In their interesting book, *Coincidences, Chaos, and All that Math Jazz*.

listing are rounded and after the first few steps digits on the right are dropped. This raises the question: Would including those dropped digits affect the results of repeated processes like this? And the answer to that question is: Yes indeed!

To demonstrate this I have carried out this same process 50 times with a calculator just as I did to obtain the table above. But I have compared those results with those obtained by the scientific program *Mathematica*, which includes many more of those far-right digits in its calculations. Here are those results, in both cases reported to 10-digit accuracy.

Iteration	Calculator results	Computer results
10	.8461071025	.8461071025
20	.7433149283	.7433149250
30	1.860800201	1.860801569
40	1.036170549	1.039438815
50	.0975808653	1.813291417

Notice how the differences in the results (they are noted in boldface) begin to accumulate until after 50 iterations the answers bear no resemblance to each other whatsoever. The lesson here is very important: *If you repeat operations many times, even with many digits of accuracy, rounding errors can accumulate and accuracy can be lost.*

I add two related points:

1. Although the results for real world problems obtained on a larger computer may well be more accurate than those of your calculator, rounding errors will accumulate on those larger computers as well.
2. You might ask when you would ever need to carry out operations like this over and over. The answer is: many times. Perhaps the best example is the calculation of interest at your local bank. Today interest is most often calculated daily and you can have calculations like 1.04^{90} for an account left for three months. That calculation is equivalent to the product $1.04 \times 1.04 \times 1.04 \dots$, with 90 of those 1.04 factors. You will learn more about this in the chapter about money.

There can be even more interesting outcomes of these kinds of accumulating effects of minor changes. The science-fiction writer Ray Bradbury is credited as the first to write about them in his 1952 story, *A Sound of Thunder*, the most often republished short story in history. In that story time-travelers find that the few changes one of them made when they traveled back in time to hunt a Tyrannosaurus rex have made changes to modern time when they return. While some of the changes are small — different spelling of words, for example — others are more telling.

Then, as so often happens, science fiction turned into science fact. In 1961, when the rapid processing of computers was first being made available to scientists, MIT mathematician Edward Lorenz set out to use his computer to predict the weather. He developed a set of equations that would process past and present weather patterns to

predict the weather of the immediate future. This is, of course, exactly the computer-based iterative process employed by meteorologists today to predict the weather.¹¹ But Lorenz, in testing his data, decided to redo his calculations. Rather than return to do them all over again, he took the data printed out part way along and entered it to continue. The result was shocking: it rapidly diverged from the original results. Here is a graph he obtained with the results of the two runs superimposed:



Figure 1.3.1. Lorenz' graphical output

At first he thought the result was a computer error, but he found that it was caused by the accumulation of minor differences exactly like those of our $x^2 - 2 \rightarrow x$ example. He realized then that long-range weather prediction would always be deeply suspect, because extremely minor changes in the original source data would generate ever greater changes over time. In speaking about what he had found, Lorenz began to use the air motion caused by a butterfly's wings as an example of tiny effects that could cause larger differences. The "Butterfly Effect" became the standard metaphor for this kind of chaos when in 1972 Philip Merilees titled his talk before the American Association for the Advancement of Science, "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?"

So think twice before you whisper to your neighbor in class: the perturbation of the air of your breath may cause a war in the Balkans.

Order

Not everything leads to chaos, however. There are times when apparent disorder leads in the opposite direction, to order. Here is an example of that kind of situation.

You may not recall the way you found the midpoint of a line segment in coordinate geometry. It is one of the easiest formulas to remember: simply take the average of the x and y values of the segment endpoints. In more formal terms, the point midway between (a, b) and (c, d) is

$$\left(\frac{a + c}{2}, \frac{b + d}{2} \right) \quad (1.3.1)$$

I use that formula for the following experiment. I locate three points on the coordinate plane: $A(0, 0)$, $B(1, 0)$ and $C(.5, 1)$. Beginning from the point A ,

¹¹Some people condemn their local weather forecasters for getting things wrong, but then rely on months-ahead predictions, like those of *The Farmer's Almanac*, that are no more than guesswork.

I choose at random one of the three points, A, B or C. I could do this by rolling a single die and letting 1 or 2 represent A; 3 or 4, B; and 5 or 6, C. Using that same random procedure, I now locate a point half-way from my current position – initially at $(0,0)$ – to that point. On the figure below, I first chose point C and located the point half-way there marked 2. You can easily calculate the coordinates of that point $(.25, .5)$ by using formula (1.3.1).

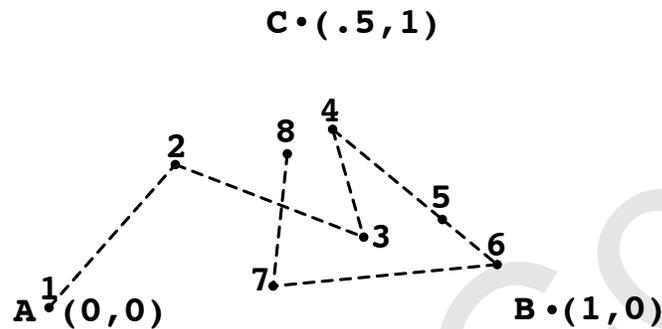


Figure 1.3.2 How the experimental points are plotted

From point 2 the random path of my example goes half-way to B, then half-way to C, then half-way to B and half-way to B again, and so on. (Note that only the points are part of the resulting configuration. The dotted segments and the labels are included only to show how those points were located.)

Now suppose you repeat that half-way procedure thousands of times placing a series of randomly chosen midpoints on the coordinate plane. You might think that you would simply fill in that triangular region ABC with points. Remarkably, the result is quite different from that expectation. The process is carried out for you in Panel 1.3.1. If you watch carefully, you will see the individual points appearing. But over time I promise you that a complex figure will appear.

Panel 1.3.1: Midpoint Repetition



Now we see again how a panel can help us carry out a routine task. I suggested that the midpoint choice could be done by tossing a die and indeed we could locate point after point that way. But using Panel 1.3.1, we let the panel do this over and over very quickly in order to generate point after point. At each step of the process the panel, in effect, chooses point A, B or C with equal chance and places the appropriate midpoint. But it is very important that you observe that, although the process is random and the placement of the points is different each time you run this panel, the resulting figure is essentially the same. Was this just a lucky outcome? You can check by running the panel several times.

This remarkable outcome mathematicians call the Sierpinski Triangle. It is named after its discoverer, the 20th century Polish mathematician, Waclaw Franciszek Sierpinski. As it happens, however, Sierpinski came upon this configuration by an entirely different method (see exercise 1.3.6).

When I first developed and ran that program, I was as shocked as I suspect you are. “Why is there that big clear triangle in the middle of the diagram?” I asked myself. “Surely I should be able to locate a midpoint in that region.” But try as hard as I could, I could not find a way of going from a point outside that region to a midpoint within it. (You should try this to convince yourself as well.)

In any case, here is an example of seeming disorder leading to a quite remarkable orderly configuration.

I note here that the rough edges you find in computer displays like this are due to the fact that the display is made up of square dots instead of dimensionless points. These dots are called pixels. There are hundreds of thousands of those pixels on the screens of today’s computers, tablets and smartphones and millions on many large screen televisions.

A middle ground between order and disorder: Fractals

You have now seen what seems like order leading to disorder and what appears at first to be disorder leading to order. Finally, I invite you to consider a kind of process that is midway between those two extremes. I will again approach this subject by means of experiments.

First, think of the following situation, which mathematicians formally call a 2-dimensional random walk. (Less formally, many refer to this as a drunkard’s walk.) We have a city laid out in square blocks cut by north-south and east-west streets. A pedestrian sets out from the center of the city for a walk along these blocks. At each corner the walker stops and randomly chooses a new direction: forward, left or right. This kind of meandering is replicated in Panel 1.3.2.

Panel 1.3.2: Random Walk 1



Run this panel several times to see different random walks that might be taken from the center of your screen. Notice that these random wanderings will soon take you to the edge of the screen, at which time the process terminates and the value of n tells you the number of blocks walked.



Panel 1.3.3: Random Walk 2

Another version of this kind of walk allows only left or right turns at each corner. Run Panel 1.3.3 several times to follow this kind of walk. Do you notice any differences between the two panels?

Those walks were completely undisciplined, but now I introduce regularity into the pedestrian’s instructions. And, because current decisions are not random but instead are based on prior actions, I can predict where this more organized person (perhaps a sober young woman this time) will go and thus use the screen more efficiently. These instructions have been built into Panel 1.3.4, which, if you set `PIXELS PER BLOCK` to 3, presents results that at first appear similar to those of the previous two panels. Watch this more disciplined pedestrian traverse the city. And notice too what happens when you run the panel again.

Panel 1.3.4: The Dragon Fractal



In Exercise 1.3.5 you will be shown more of the pattern of Panel 1.3.4. The odd shapes become more evident and would continue on a still larger screen. Their Asian appearance has given them the name Dragon Fractal or Dragon Curve.¹² Although this program follows strict instructions with no randomization built in, the figure is never quite replicated. Parts of it do look alike, but there is only what some call semi-similarity, and the next part of the figure leads to new complexity. You could, however, if you named the streets and avenues, tell exactly where you are after N moves simply by running the program and stopping after that many moves. Thus the walk is completely determined.

As its alternate name implies, this Dragon Curve is an example of a *fractal*, a kind of curve that inhabits the region between order and disorder. Fractals are a modern concept.¹³ They represent one of the most broadly applicable and most active areas of modern mathematical research. For example, you probably do not realize that those mountains the space ships pass through in films like *Star Wars* are generated by means of fractal geometry. Thus you have approached via this strange configuration the border of this fascinating subject, mathematics.

Exercises 1.3

- (1.3.1) Read Ray Bradbury's short story, *A Sound of Thunder*. (You can find it on the web by searching for "sound of thunder text". (Avoid, however, the rightly low-rated 2005 film adaptation.) What were the minor and major effects caused by the expedition?
- (1.3.2) The butterfly effect is caused by very slight initial differences in a process. Consider the repeated process $4x(1 - x) \rightarrow x$. This iteration is carried out in Panel 1.3.5. For any number you enter, the panel will repeat the process that many times and give you the result. For example, if you run it with $x = .6$, with three iterations, you will display .96, .1536 and .52002816.

Panel 1.3.5: Iterating $4x(1 - x)$

- (a) Run Panel 1.3.5 beginning with $x = .6$, iterate 30 times and record the 30th value.
- (b) Run Panel 1.3.5 beginning with $x = .5999999999$, iterate 30 times and record the 30th value.
- (c) Run Panel 1.3.5 beginning with $x = .6000000001$, iterate 30 times and record the 30th value.
- (d) How do your results compare for those very close initial values?

¹²You probably notice this odd use of the word curve. Close examination of the resulting figure shows that there are no curved lines. Despite that, the word is used in this more general sense.

¹³Just how modern? Their discoverer and the man who named them, Benoit Mandelbrot, remained active in his field until his death in 2010.

(e) How do your results relate to the butterfly effect.

(1.3.3) One thing mathematicians do when they find an interesting pattern is see if the pattern can be extended. A student raised just such a possibility: “Would the Sierpinski ‘half-way to the corner’ procedure lead to similar patterns with other polygons?” Panel 1.3.6 explores this question for a square. Run this program and describe the resulting pattern, if any.

Panel 1.3.6: Does Sierpinski Work for Squares?



(1.3.4) Run Panel 1.3.4 for $S=1$.

(a) Identify a feature that is repeated.

(b) Is that feature repeated in the same context throughout the figure?

(1.3.5) Some of the topics that have come up in this chapter represent areas of current research in mathematics. For example, the concepts of random walks are currently even being applied to financial markets. Panels 1.3.4 and 1.3.5 are two-dimensional; that is they work in a plane, but random walks apply to other dimensions as well. A random walk in one-dimension could move just up and down with the successive results recorded along a horizontal number line. You can model such a walk by flipping a coin and moving up one unit for heads, down one for tails. Such a path can then be converted to a graph as in Figure 1.3.6.

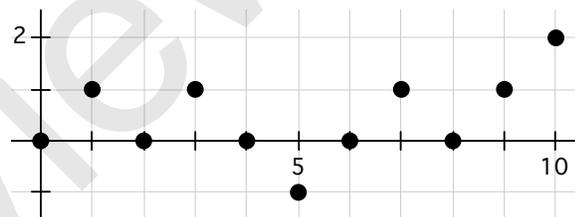


Figure 1.3.6 A Random Walk generated by the series: H T H T T H H T H H

(a) The graph in Figure 1.3.6 falls on the x -axis at many even numbered tosses. Experiment to see if you can reach the x -axis in an odd number of tosses. Try to explain your answer.

(b) There is a theorem proved with advanced mathematics that says the average distance from 0 a one-dimensional random walk reaches after N moves is $\sqrt{2N/\pi}$. Show that when $N = 25$, this value is close to 4.

(c) What is the farthest distance from the x -axis you could reach in 25 moves? Would that happen very often?

(d) What is the smallest distance from the x -axis you could reach after 25 moves? (Recall your answer to (a).)

Panel 1.3.7: One-Dimensional Random Walk



- (e) Panel 1.3.7 gathers information to test the claim in (b). It runs a session of 25 tosses, keeping track of how far from zero you end up. It then averages these distances for comparison with the value 4 you calculated in that exercise. Run Panel 1.3.7 10 times to see how your final value for 10 sessions compares with this.
- (1.3.6) Sierpinski's procedure that led to the triangle identified with his name was different from what was portrayed in this text, but his result was the same. See the *Wikipedia* entry for "Sierpinski triangle."
- (a) What is the different procedure?
 - (b) Our process does appear later, however. What is it called?
 - (c) What is the three-dimensional analog of the Sierpinski triangle called?
- (1.3.7) Access the *Wikipedia* entry for "Dragon curve." It will give you some sense of the kinds of directions to which this topic leads and some of the artistic results of those explorations.
- (a) Which of the figures do you like best and why?
 - (b) Which of the tiling examples do you find most attractive? What draws you to that particular example?



Answers for Chapter 1.

<https://url.rylanbooks.com/qGdr0>